

# Traveling solar-wind bulk-velocity fluctuations and their effects on electron heating in the inner heliosphere

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## ABSTRACT

In a recent publication, we have shown that ambient plasma electrons undergo strong heating in regions associated with compressive traveling interplanetary solar-wind bulk-velocity jumps  $\Delta U$  due to their specific interactions with the jump-inherent electric fields. After thermalization of this energy gain per shock passage through the operation of the Buneman instability, strong electron heating occurs that substantially influences the radial electron temperature profile. While our previous study describes the resulting electron temperature assuming that the amplitude of the traveling velocity jump remains constant with increasing solar distance, we now aim at a more consistent view, describing the change of the jump amplitude with distance due to the heated electrons. We describe the reduction of the jump amplitude due to energy expended by the traveling jump structure. We consider three effects; namely energy loss due to heating of electrons, energy loss due to work done against the pick-up-ion pressure gradient, and an energy gain due to nonlinear jump steepening. Taking these effects into account, we show that the decrease in jump amplitude with solar distance is more pronounced when the initial jump amplitude is higher in the inner solar system. Independent of the initial jump amplitude, it eventually decreases with increasing distance to a value of the order of  $\Delta U/U \approx 0.1$  at the position of the heliospheric termination shock, where  $\Delta U$  is the jump amplitude, and  $U$  is the average solar-wind bulk velocity. The electron temperature, on the other hand, is strongly correlated with the initial jump amplitude, leading to electron temperatures between 6000 K and 20 000 K at distances beyond 50 AU. We compare our results with in-situ measurements of the electron-core temperature from the Ulysses spacecraft in the plane of the ecliptic for  $1.5 \text{ AU} \leq r \leq 5 \text{ AU}$ , where  $r$  is the distance from the Sun. We find a very good agreement between our results and these observations, which corroborates our extrapolated predictions beyond  $r = 5 \text{ AU}$ .

**Key words.** plasmas – solar wind – Sun: heliosphere

## 1. Introduction

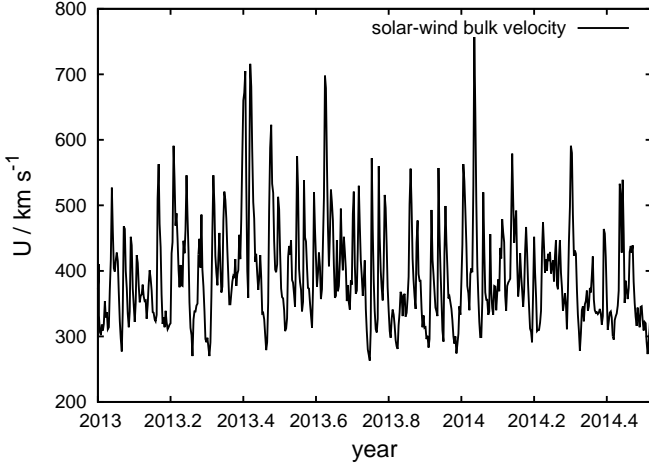
The electron temperature in the solar wind is expected to rapidly drop off with increasing distance  $r$  from the Sun, as soon as the electron heat conduction serving as the prime energy source has died out (Feldman et al. 1975; Pilipp et al. 1987; Scime et al. 1994). At distances smaller than 5 AU, electron distribution functions have been identified as core-halo structured distributions with an electron heat flux falling off with a power law according to  $\propto r^{-2.36}$  (see McComas et al. 1992). On the basis of electron data taken from the Helios, Wind, and Ulysses spacecraft, Maksimovic et al. (2005) have carefully analyzed the radial change of the core-halo-strahl structure of the electron distribution function with distance from the Sun in the range between 0.3 AU and 1.5 AU. These authors find that, while the relative abundance of core electrons remains fairly constant with distance, the relative abundance of halo electrons increases and that of strahl electrons decreases, suggesting that the relative increase in halo electrons is connected to the relative loss in strahl electrons. Interestingly enough, however, both the core electron temperature and the halo electron temperature decrease with distance. This effect can be represented by kappa distribution functions with decreasing kappa-indices and will be best fitted by the fall-off of the electron kappa index from  $\kappa = 6$  at  $r = 0.5 \text{ AU}$

to  $\kappa = 3$  at  $r = 1.5 \text{ AU}$ . The increase in the relative abundance of the halo population is interpreted as the consequence of an isotropization of the strahl population, leading to a conversion into the halo population (see also Štverák et al. 2009).

Beyond the outer ranges of the Ulysses trajectory (i.e., at solar distances beyond 5 AU), measurements of low-energy solar-wind electrons are not available. Up to now, electron temperatures have been expected to fall off to negligible values in this region for theoretical reasons. At such large distances from the Sun, processes like whistler-wave-turbulence generation due to instabilities driven by the electron heat flux (see Scime et al. 1994; Gary et al. 1994) become unimportant. Also pitch-angle scattering and energy-diffusion processes can be neglected at those distances (Schlickeiser et al. 1991; Achatz et al. 1993). However, more recently Breech et al. (2009) have presented a theoretical study of the heating of solar-wind protons and electrons by dissipation of MHD-turbulent energy. While their study shows that the theoretically obtained proton temperatures fit the Ulysses data, the theoretical electron temperatures (see Fig. 3 in Breech et al. 2009), since being too low, miss almost all the data.

As a remedy of that failure, we most recently conjectured that the interaction between electrons and the electric fields associated with traveling fluctuations in the solar-wind bulk velocity (i.e., traveling shocks) can provide an energy source for electron heating in this part of the heliosphere (Chashei & Fahr 2014). All solar-wind properties, including the solar-wind bulk veloc-

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**Fig. 1.** Solar-wind bulk velocity as a function of time in the plane of the ecliptic at 1 AU. We show OMNI-2 data (from [ftp://spdf.gsfc.nasa.gov/pub/data/omni/low\\_res\\_omni/](ftp://spdf.gsfc.nasa.gov/pub/data/omni/low_res_omni/)) from a combined set of spacecraft observations to demonstrate the ubiquitous fluctuations in the solar-wind bulk velocity.

ity  $U$ , show strongly pronounced variations on many time scales as well as shock-like structures (Feng et al. 2009; Yue & Zong 2011; Janvier et al. 2014). We show a time line of the measured solar-wind bulk velocity in the plane of the ecliptic at 1 AU in Fig. 1 (cf Echer et al. 2005; Lai et al. 2010; Sokół et al. 2013). In agreement with these observations, we find a typical occurrence rate of about 30 jumps of significant amplitude per year. These jumps are convected over the spacecraft with an average solar-wind bulk velocity of  $U \approx 400$  km/s, leading to a typical distance of about  $L_j = 3$  AU between subsequent shocks.

Fluctuations  $\Delta U(t) \equiv U(t) - \langle U(t) \rangle$ , where  $\langle \cdot \rangle$  is the time average and  $U$  is the solar-wind bulk velocity, persist to large distances from the Sun up to 20 to 40 AU as clearly demonstrated by Voyager observations (Richardson et al. 1995). Their Fig. 2 shows that, while the bulk velocity fluctuations survive up to large solar distances, the average bulk velocity  $U = \langle U(t) \rangle$  appears to be constant, implying that differential kinetic energy is not converted into kinetic energy of the wind, but into thermal degrees of freedom of the plasma system.

As mentioned in the beginning, we suspect that these bulk-velocity fluctuations are responsible for the still not well understood heating of electrons at larger distances from the Sun. We have recently proposed in a quantitative discussion that compressional bulk-velocity waves heat solar-wind electrons (Chashei & Fahr 2014). In this latter work, we determined the fraction of the differential kinetic energy of the traveling shocks and quantified the energy that is transferred to thermal energy of the solar-wind electrons by means of the Buneman instability as a function of the bulk speed  $U_2$  downstream of the velocity jump. The joint bulk speed of electrons and protons, after passing the jump-associated electric-field jump, is given by

$$m_e U_{2e} + m_p U_{2p} = (m_e + m_p) U_2, \quad (1)$$

where  $m_{e,p}$  denote the mass of the electron and of the proton, respectively, and  $U_2$  denotes the bulk velocity of the downstream center-of-mass system. With  $m_e \ll m_p$ , this expression leads to (see Chashei & Fahr 2014)

$$U_2 \approx U_{2p} + \frac{m_e}{m_p} U_{2e} \approx U_{2p} + s \sqrt{\frac{m_e}{m_p}} U_{2p}, \quad (2)$$

where  $s \equiv (U + \Delta U)/(U - \Delta U)$  is the jump compression ratio. The difference  $U_2 - U_{2p}$  is very small compared to  $U_{2p}$ , and hence the overshoot energy of the electrons in the downstream bulk frame is given by

$$\Delta W_e = \frac{1}{2} \frac{m_e m_p}{m_e + m_p} (U_{2e} - U_{2p})^2 \approx \frac{1}{2} m_e U_{2e}^2. \quad (3)$$

If this kinetic energy  $\Delta W_e$  of the overshooting electrons can be locally converted into electron heat, this process leads to an electron temperature increase  $\Delta T_e$  after each jump passage given by

$$\Delta T_e = \frac{m_e U_{2e}^2}{3k} = \frac{m_p \Delta U^2}{3k} \left(1 - \frac{1}{s^2}\right), \quad (4)$$

where  $k$  is the Boltzmann constant.

This process describes an average gain of thermal energy that leads to a systematic heating of the solar-wind electrons per radial increment  $dr$  due to repeated shock passages. The resulting radial dependence of the electron temperature can be described by a transport equation for the thermal energy. We expect that the electron heating due to accumulated jump passages in the heliosphere beyond about 5 AU is statistical in nature. We denote the average distance between consecutive jumps as  $L_j$  and define the average jump occurrence rate as  $\nu_j \equiv U/L_j$ . With these definitions, the equation for the radial electron temperature is given in the following differential form (see Chashei & Fahr 2014):

$$\frac{dT_e}{dr} + 2 \frac{T_e}{r} = \Delta T_e \frac{\Delta U}{L_j U} = \Delta T_e \frac{\Delta X}{L_j}, \quad (5)$$

where  $\Delta X \equiv \Delta U/U$ . The term on the right-hand side of Eq. (5) describes the electron heating induced by jump passages. Chashei & Fahr (2014), when solving the above equation, assumed that  $\Delta T_e = \Delta T_e(\Delta U)$  is a constant. This assumption is true if  $\Delta U$  is independent of the distance  $r$ . In that case, the radial profile of the resulting electron temperature is given by

$$T_e(x) = \frac{1}{x^2} \left( x^3 \frac{m_p U^2 \Delta X^3}{18k} \frac{r_0}{L_j} + T_{e0} \right), \quad (6)$$

where  $x \equiv r/r_0$  is the dimensionless spatial coordinate and  $T_{e0}$  denotes the electron temperature at  $r = r_0 = 1$  AU (solution shown in Fig. 1 of Chashei & Fahr 2014). This solution suffers from the inconsistency that the jump kinetic energy is assumed to be constant even after transferring energy to the electrons. In the following section of this study, we make this earlier approach more consistent by taking into account the energy consumption at the passage of each jump during this process.

## 2. Change of the Jump Amplitude with Distance from the Sun

In order to increase the consistency of our approach, we now include higher-order corrections to the electron heating due to the variation of the jump amplitude  $\Delta U$  with distance  $r$ . This amplitude is assumed to be the primary physical reason for the gain of thermal energy of the electrons. Therefore, we have to describe the change of  $\Delta U$  due to energy expended by the excess kinetic energy of the jump structure adequately. In-situ observations by the Voyager-2 spacecraft at distances between 10 AU and 40 AU from the Sun (Richardson et al. 1995) show that, compared to solar-wind bulk-velocity measurements carried out simultaneously at 1 AU by IMP-8, the average solar-wind speed does not change with distance. On the other hand, the amplitude of the

speed fluctuations strongly decreases with distance from the Sun (see Fig. 6 of Richardson et al. 1995). This observation indicates that these fluctuations do work, while the bulk-solar-wind outflow does not. Our theoretical approach is based on these observations, adopting that the average solar-wind speed  $U$  is constant with distance from the Sun. Based on this observationally supported assumption, we consider three effects that determine the change of  $\Delta U$  with distance  $r$ :

- heating of electrons,
- work done against the slower side of the jump with its higher pick-up-ion pressure, and
- steepening of the jump profile by nonlinear superpositions of small-scale bulk-velocity fluctuations.

In the following we shall separately look into these three different effects.

### 2.1. a) Reduced Compression due to Electron Heating

We consider the spatial divergence of the jump-associated flow of excess kinetic energy on the high-velocity side of a jump with the amplitude  $\Delta X = \Delta U/U$ . This jump acts as a local source of electron thermal energy, and this heating reflects a local energy sink for the excess kinetic energy that is represented by the compression profile  $\Delta X(r)$ . Using Eq. (5) for the electron temperature, we can formulate an expression for the energy sink associated with this jump as the divergence of the excess kinetic energy flow:

$$\frac{1}{r^2} \frac{d}{dr} r^2 \left( U \frac{1}{2} n_e m_p \Delta U^2 \right) = -\frac{3}{2} U k \Delta T_e n_e \frac{\Delta U}{L_j U}, \quad (7)$$

where  $n_e = n_p = n$  is the local solar wind electron/proton number density and  $\Delta T_e$  is the electron temperature increase per jump passage as given by Eq. (4). We assume that the mean bulk velocity  $U = (1/2)(U + \Delta U + U - \Delta U)$  is constant and find

$$\frac{d}{dr} (\Delta U^2 n_e) + \frac{2}{r} (\Delta U^2 n_e) = -3 \frac{k \Delta T_e \Delta U}{m_p U L_j}. \quad (8)$$

Supported by Voyager data, we assume that the traveling jumps in bulk velocity have a small amplitude ( $\Delta U \ll U$ , which is equivalent to  $\Delta X \ll 1$ ). This observation allows us to approximate the electron-temperature increase per jump passage in Eq. (4) using the linearizations

$$s = \frac{1 + \Delta X}{1 - \Delta X} \simeq 1 + 2 \Delta X \quad (9)$$

and

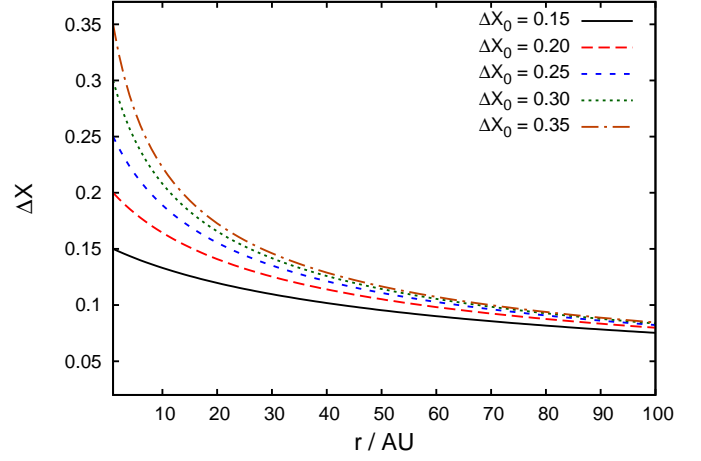
$$s^2 \simeq (1 + 2 \Delta X)^2 \simeq 1 + 4 \Delta X. \quad (10)$$

We can then rewrite Eq. (4) as

$$\Delta T_e \simeq \frac{m_p \Delta U^2}{3k} [1 - (1 - 4 \Delta X)] = \frac{4 m_p U^2 \Delta X^3}{3k} \quad (11)$$

and obtain from Eq. (8)

$$\frac{2}{\Delta X} \frac{d \Delta X}{dr} + \frac{1}{n_e} \frac{dn_e}{dr} + \frac{2}{r} = -4 \frac{\Delta X^2}{L_j}. \quad (12)$$



**Fig. 2.** Compression  $\Delta X$  as a function of distance  $r$  from the Sun for five different values of  $\Delta X_0$  at  $r_0 = 1$  AU with  $r_0/L_j = 1/3$ . The compression decreases with distance and approaches a value of about 0.1 at the position of the termination shock. The dependence on the jump occurrence  $r_0/L_j$  is discussed in Chashei & Fahr (2014).

Assuming a spherically symmetric decrease in density of the average solar wind flow with  $n_e \propto r^{-2}$ , we then obtain

$$\frac{2}{\Delta X} \frac{d \Delta X}{dr} = -4 \frac{\Delta X^2}{L_j} \quad (13)$$

and find

$$\frac{d \Delta X^{-2}}{dr} = 4 \frac{1}{L_j}. \quad (14)$$

From this relation, we derive in a first step

$$|\Delta X^{-2}|_{r_0}^r = 4 \frac{1}{L_j} \int_{r_0}^r dr = \frac{4}{L_j} (r - r_0), \quad (15)$$

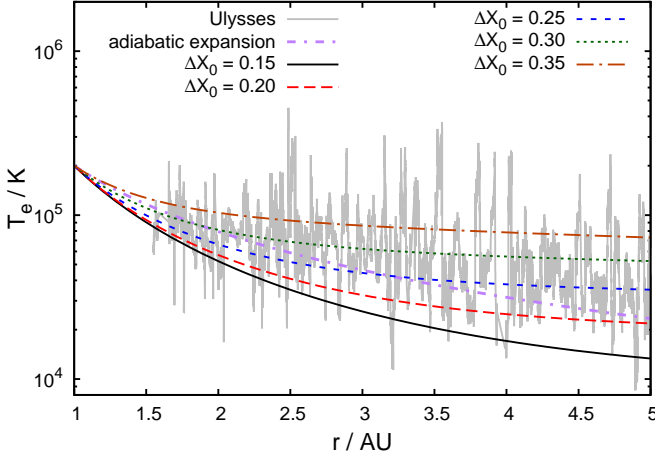
which delivers a solution of the form

$$\Delta X = \frac{1}{\sqrt{\Delta X_0^{-2} + \frac{4}{L_j} (r - r_0)}} = \frac{\Delta X_0}{\sqrt{1 + \frac{4 r_0}{L_j} \Delta X_0^2 (x - 1)}}. \quad (16)$$

We show  $\Delta X$  as a function of distance  $r$  from the Sun for five different values of  $\Delta X_0$  in Fig. 2. We use  $r_0/L_j = 1/3$  in agreement with observations at 1 AU (see Fig. 1). The jump amplitude  $\Delta X$  remarkably decreases with increasing distance from the Sun. The decrease is even more pronounced for cases in which the initial value  $\Delta X_0$  is higher in the inner solar system. However, independent of  $\Delta X_0$ , the jump amplitude assumes values of  $\lesssim 0.1$  at the position of the heliospheric termination shock (i.e., at  $r \approx 90$  AU).

With this result for the dependence of  $\Delta X$  on  $r$ , we solve the earlier differential equation (5) for the resulting electron temperature and obtain

$$\frac{dT_e}{dx} + 2 \frac{T_e}{x} = \frac{4}{3} \frac{m_p U^2}{k \lambda} \frac{\Delta X_0^4}{(1 + 4 \Delta X_0^2 \frac{x-1}{\lambda})^2}, \quad (17)$$



**Fig. 3.** Electron temperature  $T_e$  as a function of distance  $r$  from the Sun for five different values of  $\Delta X_0$  with  $r_0/L_j = 1/3$  and  $U = 400$  km/s. The electron temperature is greater than adiabatic. In addition, we show Ulysses observations of the in-situ electron-core temperature in the plane of the ecliptic and the prediction from adiabatic expansion.

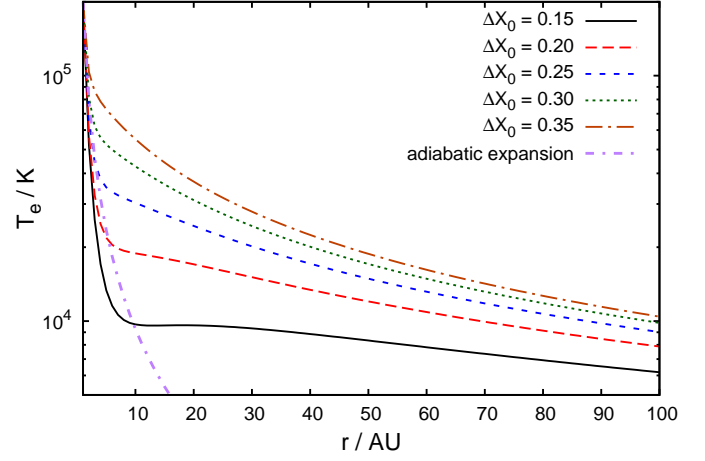
where  $\lambda \equiv L_j/r_0$ . The solution of this inhomogeneous differential equation is given by

$$T_e(x) = \frac{1}{x^2} \left\{ \frac{4}{3} \frac{m_p U^2}{k \lambda} \Delta X_0^4 \int_1^x \frac{y^2}{[1 + a(x-1)]^2} dy + T_{e0} \right\} \\ = \frac{1}{x^2} \left\{ \frac{4}{3} \frac{m_p U^2}{k \lambda a^3} \Delta X_0^4 \left[ 2(a-1) \ln[1 + a(x-1)] + \frac{a(x-1)[1 + a(x-1) + (a-1)^2]}{1 + a(x-1)} \right] + T_{e0} \right\} \quad (18)$$

with  $a = 4\Delta X_0^2/\lambda$ . We show the result of Eq. (18) for five different values of  $\Delta X_0$  in Figs. 3 and 4. We use  $T_{e0} = 2 \times 10^5$  K and  $U = 400$  km/s. In Fig. 3, we show our results for  $1.5 \text{ AU} \leq r \leq 5 \text{ AU}$  and compare them with in-situ Ulysses measurements of the electron-core temperature during the spacecraft's first orbit in the plane of the ecliptic (from December 28, 1990 until December 31, 1991). The data was taken with Ulysses' SWOOPS experiment (Bame et al. 1992). The comparison between our predictions and in-situ measurements shows a very good agreement between theory and observation. The modeled and observed electron temperatures are significantly higher than predicted for an adiabatically-expanding gas (i.e.,  $T_e \propto r^{-4/3}$ ). We achieve the best agreement for values of  $\Delta X_0$  between 0.25 and 0.3. In Fig. 4, we extrapolate our results beyond 5 AU and show our predictions for  $1 \text{ AU} \leq r \leq 100 \text{ AU}$ . The electron temperature in our model is significantly higher than predicted according to adiabatic expansion beyond 10 AU for all shown values of  $\Delta X_0$ . It assumes values of about  $T_e \approx 10^4$  K at the position of the heliospheric termination shock.

## 2.2. b) Change of Compression due to Work done against Entropized Pick-Up Ions

In this section, we consider another effect that may also contribute to a decrease in the jump amplitude  $\Delta X$ , namely the work done by the faster front against the difference in ion pressure over the traveling shock front. The faster regime ( $U_1 = U + \Delta U$ )



**Fig. 4.** Electron temperature  $T_e$  as a function of distance  $r$  from the Sun for five different values of  $\Delta X_0$  with  $r_0/L_j = 1/3$  and  $U = 400$  km/s. The electron temperature beyond 10 AU is higher than predicted according to adiabatic expansion.

is running into the slower regime ( $U_2 = U - \Delta U$ ) with a differential velocity  $\Delta U$ . During this process, the plasma has to do work against the pressure difference between the two regimes in order to adapt the flow to the slower regime (i.e.,  $U_2$ ). We estimate the work done per unit time as

$$\frac{d\epsilon(s)}{dt} = -\frac{\Delta U \cdot \Delta P}{D} = -\Delta U \frac{P_2^* - P_1^*}{D}, \quad (19)$$

where  $D$  is the transit distance over the shock profile, and  $P_{1,2}^*$  are the adaptive pressures (i.e., the entropized kinetic energy densities) on the upstream and on the downstream sides of the jump, respectively. Especially in the outer heliosphere ( $r \geq 5 \text{ AU}$ ), the ion pressure is dominated by the pick-up-ion pressures on either side of the jump. Under this assumption, the expressions for the ion pressures are significantly simplified (see Fahr et al. 2012) for a perpendicular jump ( $\Delta U \perp \mathbf{B}$ ), leading to

$$P_2^* - P_1^* \approx P_{1,\text{pui}} \left[ \frac{s}{3} \left( 2A_\perp(s) + \frac{s^2}{A_\perp^2(s)} \right) - 1 \right], \quad (20)$$

where the remaining pressure adaptation function  $A_\perp(s)$  in case of a perpendicular shock is simply given by  $A_\perp(s) = s$  with  $s \approx 1 + 2\Delta X$ . This leads to

$$P_2^* - P_1^* \approx P_{1,\text{pui}} \left[ \frac{s}{3} (2s + 1) - 1 \right] \\ = P_{1,\text{pui}} \left[ \frac{1 + 2\Delta X}{3} (3 + 4\Delta X) - 1 \right] \approx \frac{10}{3} \Delta X P_{1,\text{pui}}, \quad (21)$$

which allows us to formulate the ion-induced energy change as

$$\frac{d\epsilon(s)}{dt} = -\frac{\Delta U \cdot \Delta P}{D} = -\frac{10}{3} \frac{\Delta U}{D} \Delta X P_{1,\text{pui}} \\ = -\frac{10}{3} \frac{U \Delta X^2}{D} P_{1,\text{pui}}. \quad (22)$$

With this additional term, we now obtain the following corrected differential equation for  $\Delta X$ :

$$\frac{2}{\Delta X} \frac{d\Delta X}{dr} + \frac{1}{n_e} \frac{dn_e}{dr} + \frac{2}{r} = -4 \frac{\Delta X^2}{L_j} - \frac{20}{3} \frac{P_{1,\text{pui}}}{n_e m_p U^2 D}. \quad (23)$$

We write the upstream pick-up-ion pressure in the form  $P_{1,\text{pui}} = \zeta n_e k T_{\text{pui}}$  with the pick-up-ion abundance ratio  $\zeta = n_{\text{pui}}/n_e$ . This leads to the new differential equation

$$\frac{2}{\Delta X} \frac{d\Delta X}{dr} + \frac{1}{n_e} \frac{dn_e}{dr} + \frac{2}{r} = -4 \frac{\Delta X^2}{L_j} - \frac{20}{3} \frac{1}{m_p U^2 D} \zeta k T_{\text{pui}}. \quad (24)$$

Again, taking  $n_e \propto r^{-2}$ , this relation then simplifies to

$$\frac{2}{\Delta X} \frac{d\Delta X}{dr} = -4 \frac{\Delta X^2}{L_j} - \frac{20}{3} \frac{1}{D} \frac{\zeta k T_{\text{pui}}}{m_p U^2}. \quad (25)$$

We introduce the pick-up-ion Mach number  $M_{\text{pui}}^2 \equiv m_p U^2 / \zeta k T_{\text{pui}} \simeq 1$  and assume that this number be constant in the outer heliosphere (see e.g. Fahr & Ruciński 1999; Fahr 2007). We then find

$$\Delta X = \frac{1}{\sqrt{\left(\frac{3DM_{\text{pui}}^2}{5L_j} + \frac{1}{\Delta X_0}\right) \exp\left[\frac{20}{3} \frac{(r-r_0)}{DM_{\text{pui}}^2}\right] - \frac{3DM_{\text{pui}}^2}{5L_j}}} = \frac{\Delta X_0}{\sqrt{\left(\frac{3DM_{\text{pui}}^2}{5L_j} \Delta X_0^2 + 1\right) \exp\left[\frac{20}{3} \frac{(x-1)r_0}{M_{\text{pui}}^2 D}\right] - \frac{3DM_{\text{pui}}^2}{5L_j} \Delta X_0^2}}. \quad (26)$$

Expansion of the exponential term in Eq. (26) for  $D \ll r_0$  leads to Eq. (16). Since the shock transit distance is much smaller than 1 AU, the corrections due to the pick-up-ion pressure lead to qualitatively very similar curves as already shown in Fig. 2.

### 2.3. c) Increased Compression due to Nonlinear Wave Steepening

One may also have to envisage processes that counteract the processes described in a) and b), namely processes that support a pile-up of the bulk-velocity jump amplitude. For instance, fluctuations in the bulk velocity may cause such a pile-up by nonlinear superposition. Therefore, we consider wave steepening in the system in addition to the previously discussed processes. Small-scale velocity fluctuations described by  $\delta U(x, t) = \delta U(k) \cos[k(x - Ut)]$  can pile up into a large-scale fluctuation with  $L \simeq L_j = 2\pi/k_{\min}$  due to nonlinear wave-coupling and dissipation processes. For one-dimensional waves, this situation is described by the following equation (see Infeld & Rowlands (1990), pp. 6-10 or Treumann & Baumjohann (1997), pp. 244-280):

$$\frac{\partial}{\partial t} \delta U + \delta U \frac{\partial}{\partial x} \delta U = F, \quad (27)$$

where  $F$  denotes a dissipation force that counteracts the nonlinear term on the left-hand side and compensates for catastrophic wave steepening and wave breaking. In case of the so-called Burger's equation (see Treumann & Baumjohann 1997), a particular dissipative force is introduced in place of  $F$  that is proportional to the second derivative of the velocity perturbation, leading to the following differential equation:

$$\frac{\partial}{\partial t} \delta U + \delta U \frac{\partial}{\partial x} \delta U = \alpha \frac{\partial^2}{\partial x^2} \delta U, \quad (28)$$

where  $\alpha$  is a positive dissipation coefficient that acts like a diffusion coefficient (assumed to be constant with distance  $r$ ). The background plasma moves with the velocity  $U$ , and  $\delta U$  represents the superposition  $\delta U = U + \Delta U$ . If the nonlinear steepening

of  $\delta U$  (second term on the left-hand side of the above Burger's equation (28)) increases, the dissipative term on the right-hand side can compensate for the nonlinear term and can allow for a stationary solution in the system co-moving with the nonlinear wave profile. We assume that this developing nonlinear wave asymptotically moves with the velocity  $\Delta U$ . This allows us to write the Burger's equation in this particular co-moving system, where the first term of the left-hand side disappears (i.e.,  $\partial \delta U / \partial t = 0$ ) when we transform the equation to space coordinates  $y = x - \Delta U t$ . This procedure then leads to

$$(\delta U - \Delta U) \frac{\partial}{\partial y} \delta U = \alpha \frac{\partial^2}{\partial y^2} \delta U. \quad (29)$$

The solution of this equation is easily obtained in the form of a velocity shock ramp given by

$$\delta U - \Delta U = -\Delta U \tanh\left(\frac{\Delta U y}{2\alpha}\right), \quad (30)$$

which can be rewritten in the form

$$\delta U = \Delta U \left[ 1 - \tanh\left(\frac{\Delta U y}{2\alpha}\right) \right]. \quad (31)$$

In order to estimate the appropriate value of  $\alpha$  (which has the dimension of  $\text{cm}^2/\text{s}$ ), we return to the original Burger's equation and estimate the time scale for steepening (or in the opposite case: for dissolution) of the wave profile by the pure diffusion-type equation (i.e., domination of the dissipation term) given by

$$\frac{\partial}{\partial t} \delta U = \alpha \frac{\partial^2}{\partial x^2} \delta U. \quad (32)$$

We find the solution of this equation within the system  $[-D; +D]$  by

$$\delta U(x, t) = \delta U_0 \frac{2D}{\sqrt{4\pi\alpha t}} \exp\left(-\frac{x^2}{4\alpha t}\right). \quad (33)$$

The kinetic-energy density of the velocity fluctuations within the two flanks  $[-D; +D]$  of such a velocity structure with the structure scale  $D$  is given by

$$\epsilon_{\text{nl}} = \frac{1}{2D} \int_{-D}^{+D} \frac{1}{2} m_p n_e \delta U^2 dx. \quad (34)$$

When free diffusion would operate, its temporal change is given by

$$\dot{\epsilon}_{\text{nl}} = \frac{1}{2D} \frac{d}{dt} \int_{-D}^{+D} \frac{1}{2} m_p n_e \delta U^2 dx. \quad (35)$$

Taking the above expression for  $\epsilon_{\text{nl}}$  for nonlinear diffusion or steepening per unit volume, we obtain

$$\begin{aligned} \dot{\epsilon}_{\text{nl}} &= \frac{1}{2D} \frac{1}{2} m_p n_e \delta U_0^2 \frac{d}{dt} \left[ \frac{4D^2}{4\pi\alpha t} \int_{-D}^{+D} \exp\left(-\frac{2x^2}{4\alpha t}\right) dx \right] \\ &= D m_p n_e \delta U_0^2 \frac{d}{dt} \left[ \frac{\sqrt{2\alpha t}}{4\pi\alpha t} \int_{-D/\sqrt{2\alpha t}}^{D/\sqrt{2\alpha t}} \exp(-y^2) dy \right]. \end{aligned} \quad (36)$$

Evaluating this integral expression furtheron leads to

$$\begin{aligned}\dot{\epsilon}_{\text{nl}} &= Dm_p n_e \delta U_0^2 \frac{d}{dt} \left[ \frac{\text{erf}\left(\frac{D}{\sqrt{2\alpha t}}\right)}{\sqrt{8\pi\alpha t}} \right] \\ &= Dm_p n_e \delta U_0^2 \left[ \frac{e^{-D^2/2\alpha t} \frac{d}{dt} \frac{D}{\sqrt{2\alpha t}}}{\pi \sqrt{8\alpha t}} - \frac{1}{2} \frac{\text{erf}\left(\frac{D}{\sqrt{2\alpha t}}\right)}{(8\pi\alpha t)^{3/2}} 8\pi\alpha \right],\end{aligned}\quad (37)$$

and finally to

$$\dot{\epsilon}_{\text{nl}} = \frac{Dm_p n_e \delta U_0^2}{\sqrt{8\pi\alpha t}} \left[ \frac{2\alpha D e^{-D^2/2\alpha t}}{(8\pi\alpha t)^{3/2}} - \frac{\text{erf}\left(\frac{D}{\sqrt{2\alpha t}}\right)}{2t} \right]. \quad (38)$$

According to the profile in Eq. (31),  $D \approx \alpha/\Delta U$ . On the other hand, the characteristic time  $\tau$  of the shock passage is given by  $\tau = D/\Delta U = \alpha/\Delta U^2$ . Evaluating now the above expression for this characteristic time  $\tau$  leads to the following expression:

$$\dot{\epsilon}_{\text{nl}} = \frac{Dm_p n_e \delta U_0^2}{\sqrt{8\pi D^2}} \left[ \frac{2D^2 \Delta U e^{-D^2/2D^2}}{(8\pi D^2)^{3/2}} - \Delta U \frac{\text{erf}\left(\frac{D}{\sqrt{2D^2}}\right)}{2D} \right], \quad (39)$$

or finally (with  $\Delta U \approx \delta U_0$  as suggested by Eq. (33)) to

$$\dot{\epsilon}_{\text{nl}} = \frac{m_p n_e \Delta U^3}{\sqrt{8\pi D}} \left[ \frac{2e^{-1/2}}{(8\pi)^{3/2}} - \frac{1}{2} \text{erf}\left(\frac{1}{\sqrt{2}}\right) \right]. \quad (40)$$

We then obtain the following transport equation with the newly found term for  $\dot{\epsilon}_{\text{nl}}$ :

$$\frac{1}{r^2} \frac{d}{dr} r^2 \left( U \frac{1}{2} n_e m_p \Delta U^2 \right) = -\frac{3}{2} U k \Delta T_e n_e \frac{\Delta U}{U L_j} - \dot{\epsilon}_{\text{nl}} (\Delta U). \quad (41)$$

The last term on the right-hand side represents the energy that is required in order to maintain the jump profile. Free diffusion would instead destroy the profile according to Eq. (35). The transport equation is then given by

$$\begin{aligned}\frac{1}{r^2} \frac{d}{dr} r^2 \left( U \frac{1}{2} n_e m_p \Delta U^2 \right) &= -\frac{3}{2} U k \Delta T_e n_e \frac{\Delta U}{U L_j} \\ &\quad - \frac{m_p n_e \Delta U^3}{\sqrt{8\pi D}} \left[ \frac{2e^{-1/2}}{(8\pi)^{3/2}} - \frac{1}{2} \text{erf}\left(\frac{1}{\sqrt{2}}\right) \right].\end{aligned}\quad (42)$$

With the definition

$$\Gamma \equiv \left[ \frac{1}{2} \text{erf}\left(\frac{1}{\sqrt{2}}\right) - \frac{2e^{-1/2}}{(8\pi)^{3/2}} \right] \approx 0.3, \quad (43)$$

we find

$$\frac{d}{dr} (\Delta U^2 n_e) + \frac{2}{r} (\Delta U^2 n_e) = -3 \frac{k \Delta T_e n_e}{m_p} \frac{\Delta U}{U L_j} + \frac{2n_e \Delta U^3}{\sqrt{8\pi D U}} \Gamma. \quad (44)$$

Insertion of  $\Delta T_e$  from Eq. (11) then leads to the following differential equation:

$$\frac{d}{dr} (\Delta U^2 n_e) + \frac{2}{r} (\Delta U^2 n_e) = -\frac{4U^2 n_e \Delta X^4}{L_j} + \frac{2n_e \Delta U^3}{\sqrt{8\pi D U}} \Gamma, \quad (45)$$

or equivalently to

$$\frac{d}{dr} \Delta X^2 + \Delta X^2 \frac{1}{n_e} \frac{dn_e}{dr} + \frac{2}{r} \Delta X^2 = -\frac{4\Delta X^4}{L_j} + \frac{2\Delta X^3}{\sqrt{8\pi D}} \Gamma \quad (46)$$

and

$$\frac{2}{\Delta X} \frac{d}{dr} \Delta X + \frac{1}{n_e} \frac{dn_e}{dr} + \frac{2}{r} = -\frac{4\Delta X^2}{L_j} + \frac{2\Delta X}{\sqrt{8\pi D}} \Gamma. \quad (47)$$

For a radially symmetric density drop-off, we rewrite the transport equation including terms that decrease (first term) and that increase (second term) the compression as

$$\frac{2}{\Delta X} \frac{d}{dr} \Delta X = -\frac{4\Delta X^2}{L_j} + \frac{2\Delta X}{\sqrt{8\pi D}} \Gamma. \quad (48)$$

The combination of decreasing and increasing factors can lead to a vanishing gradient and thus a constant compression  $\Delta X$  if

$$\Delta X = \frac{\Gamma}{2\sqrt{8\pi}} \frac{L_j}{D} \approx \frac{0.3}{10} \frac{L_j}{D} = 3 \times 10^{-2} \frac{L_j}{D}. \quad (49)$$

Therefore, the newly derived term for structure steepening will only compete with the first term in case the jump amplitude has dropped down to values of  $\Delta X \lesssim 10^{-2} (L_j/D)$ . Looking for a solution of the full equation, we start from the solution of

$$\frac{2}{\Delta X} \frac{d}{dr} \Delta X = -\frac{4\Delta X^2}{L_j}. \quad (50)$$

As shown in Section 2.1, the solution is given by

$$\Delta X = \frac{\Delta X_0}{\sqrt{1 + \frac{4}{L_j} \Delta X_0^2 (r - r_0)}}. \quad (51)$$

The solution of the other part,

$$\frac{2}{\Delta X} \frac{d}{dr} \Delta X = \frac{2\Delta X}{\sqrt{8\pi D}} \Gamma, \quad (52)$$

is derived from

$$\frac{1}{\Delta X^2} \frac{d}{dr} \Delta X = -\frac{d}{dr} \Delta X^{-1} = \frac{1}{\sqrt{8\pi D}} \Gamma \quad (53)$$

and yields

$$\Delta X^{-1} - \Delta X_0^{-1} = -\frac{1}{\sqrt{8\pi D}} \Gamma (r - r_0). \quad (54)$$

This leads to

$$\Delta X = \frac{1}{\Delta X_0^{-1} - \frac{\Gamma}{\sqrt{8\pi D}} (r - r_0)} = \frac{\Delta X_0}{1 - \frac{\Gamma \Delta X_0}{\sqrt{8\pi D}} (r - r_0)}. \quad (55)$$

According to these considerations, the general solution is given by the superposition

$$\Delta X = \frac{\Omega \Delta X_0}{\sqrt{1 + \frac{4r_0}{L_j} \Delta X_0^2 (x - 1)}} + \frac{\Phi \Delta X_0}{1 - \frac{\Gamma \Delta X_0}{\sqrt{8\pi}} \frac{r_0}{D} (x - 1)}. \quad (56)$$

The corrections due to nonlinear wave steepening are small as long as  $\Delta X \gtrsim 10^{-2}$ . In those cases, we only need to consider the first term and hence retain the earlier solution which we derive in Sect. 2.1 and display in Figs. 3 and 4.

### 3. Conclusions

We have shown that traveling solar-wind bulk-velocity jumps effectively process solar-wind electrons in energy at their propagation outwards from the Sun through the inner heliosphere. These fluctuations in the solar-wind bulk velocity are ubiquitous as shown in Fig. 1. In an earlier paper, we have shown that this energization can be expressed in terms of a substantial temperature increase of the solar-wind electrons at larger distances from the Sun of about 50 AU to 90 AU. Assuming that the jump amplitude  $\Delta X = \Delta U/U$  of the propagating bulk-velocity structure does not change with solar distance  $r$ , the previous study predicts electron temperatures of more than  $10^5$  K at 90 AU (i.e., at about position of the solar-wind termination shock). In this study, we introduce higher-order corrections due to the fact that the energy for the energization of solar-wind electrons is taken from the kinetic excess energy of the propagating jump structure. We find that the previous assumption of a constant jump amplitude  $\Delta X$  is most probably unrealistic. In addition, such jump structures do permanently work against the ion excess pressure on the downstream side of the shock structure. Taking into account these two physical processes allows us to quantitatively show how the jump amplitude  $\Delta X = \Delta X(r)$  decreases with distance from the Sun, eventually reducing  $\Delta X$  independent of the initial value  $\Delta X_0$  of the jump amplitude to values of  $\lesssim 0.1$  at the termination shock as shown in Fig. 2. The nonlinear pile-up of bulk-velocity fluctuations counteracts these two mechanisms with the tendency to reform the solitary jump structure by forming waves at larger scales. We find, however, that this mechanism is most likely not effective enough to compensate for the reduction of  $\Delta X$  with distance, unless  $\Delta X \lesssim 10^{-2}$ .

Although the described mechanisms lead to a reduction of  $\Delta X$  with distance from the Sun as shown in Fig. 2, the jump-induced heating mechanism still leads to higher electron temperatures than anticipated due to adiabatic cooling at solar distances beyond 10 AU. We predict values above 6000 K to 20 000 K (strongly depending on the initial value of the jump amplitude  $\Delta X_0$  in the innermost heliosphere at  $r = r_0 = 1$  AU) at distances beyond 50 AU with the solar-wind electron-temperature profiles  $T_e(r)$  shown in Figs. 3 and 4. Our results show a very good agreement with in-situ measurements of the electron-core temperature in the plane of the ecliptic from the Ulysses spacecraft. We achieve the best agreement for values of  $\Delta X_0$  between 0.25 and 0.3, suggesting that these values describe the realistic initial jump amplitude in the plane of the ecliptic. In-situ observations of the electron temperature are not available for heliocentric distances beyond 5 AU, so that our results are a predictive extrapolation beyond the explored range.

We conclude that solar-wind electrons do not rapidly cool off with distance from the Sun as it has been generally assumed up to now. They cannot be considered cold beyond 10 AU. Instead, they need to be considered keeping track with the solar wind ion temperatures at large distances (see Richardson et al. 1995).

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